Enrollment No: ______________________ **Exam Seat No:**______________________

C.U.SHAH UNIVERSITY

Summer Examination-2016

Subject Name : Group Theory

Subject Code : 5SC04GPE1 **Branch: M.Sc. (Mathematics)**

Semester : 4 **Date : 16/05/2016** **Time : 02:30 To 05:30** **Marks : 70**

Instructions:

(1) Use of Programmable calculator and any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

SECTION – I

Q-1 **Attempt the Following questions** **(07)**

a. Show that the mapping $\psi: (\mathbb{R}_+; \cdot) \to (\mathbb{R}; +)$, where $\psi(x) = \log x$, $x \in \mathbb{R}_+$ is an isomorphism. **(02)**

b. Express $(2\ 1\ 3\ 4\ 5\ 6)(1\ 7\ 2)$ of S_7 as composition of disjoint cycles. **(02)**

c. Find only two non-isomorphic abelian groups of order 360. **(02)**

d. Define: simple group. **(01)**

Q-2 **Attempt all questions** **(14)**

a. Let G be a group of order 108. Show that there exists a normal subgroup of order 27 or 9. **(07)**

b. For a fixed element g of a given group G, if $i_g: G \to G$, where $i_g(x) = gxg^{-1}$, $x \in G$, show that **(07)**
(i) $i_g \in Aut(G)$,
(ii) Set $I(G) = \{i_g : g \in G\}$ is a normal subgroup of $Aut(G)$.

OR

Q-2 **a.** Obtain $Aut(G)$ when $G = V_4$ −Klein's Four group. **(07)**

b. State and prove Cayley's theorem. **(07)**

Q-3 **Attempt all questions** **(14)**

a. If the order of a finite group G is divisible by a prime number p, show that G has an element of order p. **(07)**

b. Prove that each permutation $f \in S_n$ can be expressed as a composition of disjoint cycles. **(07)**

OR

Q-3 **a.** Let G be a group, show that **(07)**
(i) The set of conjugate class of G is a partition of G.
(ii) $|C(a)| = [G: N(a)]$.
(iii) If G is finite, $|G| = \sum[G: N(a)]$, a running over exactly one element from each conjugate class.

b. Show that for $n \geq 2$, the set A_n of even permutation in S_n is a subgroup of order $\frac{n!}{2}$. **(07)**

SECTION – II

Q-4 **Attempt the Following questions** **(07)**

a. Is group of order 121 abelian? Justify your answer! **(02)**
b. Define: action of group G on a set X. **(02)**
c. Find the group of automorphisms of $(Z, +)$. **(02)**
d. Define: orbit of group. **(01)**

Q-5 **Attempt all questions** **(14)**

a. Let G be a finite group, and let p be a prime. If p^m divides $|G|$, then show that G has a subgroup of order p^m. **(07)**
b. If a permutation $f \in S_n$ has t and s number of transpositions in two representations as composition of transpositions, then show that t and s are either both odd or both even. **(07)**

OR

Q-5 **a.** Let G be a group of order pq, where p and q are prime numbers such that $p > q$ and $q \nmid (p-1)$, then show that G is cyclic. **(07)**
b. Show that the order of a permutation $f \in S_n$ is the least common multiple of the lengths of its disjoint cycles. **(07)**

Q-6 **Attempt all questions** **(14)**

a. Define: permutation of a group. Show that for $n \geq 3$, each element in A_n can be expressed as a composition of three cycles. **(07)**
b. If G is a group with $Aut(G) = \{I_G\}$, then show that **(07)**
(i) G is commutative,
(ii) $a^2 = e$ for each $a \in G$.

OR

Q-6 **a.** (i) Show that a group of order 1986 is not simple, **(07)**
(ii) If the order of a group is 42, then prove that its Sylow $7-$subgroup is normal.
b. Let G be a finite group, and let p be a prime, then prove that all Sylow p-subgroups of G are conjugate, and their number n_p divides $O(G)$ and satisfies $n_p \equiv 1 (mod\ p)$. **(07)**